

Sleganje plitkih temelja

1. primer

Pravougaoni temelj dimenzija $B/L=2/4\text{m}$ je fundiran na dubini $D_f=1\text{m}$ i opterećen jednakopodeljenim opterećenjem $q=150\text{kN/m}^2$. Teren je debljine $H=9\text{m}$, podzemna voda je na dubini $NPV=3.0\text{m}$, zapreminska težina tla iznad NPV je $\gamma_t=17.5\text{kN/m}^3$, ispod NPV je $\gamma'=11\text{kN/m}^3$, a modul stišljivosti tla je $M_v=8.5\text{MN/m}^2$. Duž osovine temelja, u tačkama na dubini $z=0, 2, 4, 6, 8\text{m}$ sračunati geostatičke efektivne vertikalne napone, priraštaj vertikalnih napona usled opterećenja primenom metode ugla rasprostiranja napona za $\tan\alpha=0.5$, metode Štajnbrenera i metode Kanija, deformacije za priraštaj napona prema metodi ugla rasprostiranja, kao i sleganje analitičkom integracijom deformacija, numeričkom integracijom prema trapeznom pravilu i prema Simpsonovom pravilu.

Temelj	$B := 2 \text{ m}$	$L := 4 \text{ m}$	$D_f := 1 \text{ m}$	$q := 150 \frac{\text{kN}}{\text{m}^2}$	
Tlo	$H := 9 \text{ m}$	$NPV := 3 \text{ m}$	$\gamma_t := 17.5 \frac{\text{kN}}{\text{m}^3}$	$\gamma' := 11 \frac{\text{kN}}{\text{m}^3}$	$M_v := 8.5 \frac{\text{MN}}{\text{m}^2}$
Dubine	$z_0 := 0 \text{ m}$	$z_1 := 2 \text{ m}$	$z_2 := 4 \text{ m}$	$z_3 := 6 \text{ m}$	$z_4 := 8 \text{ m}$
	$\Delta z := 2 \text{ m}$	$D_f = z_0$	$NPV = z_1$		

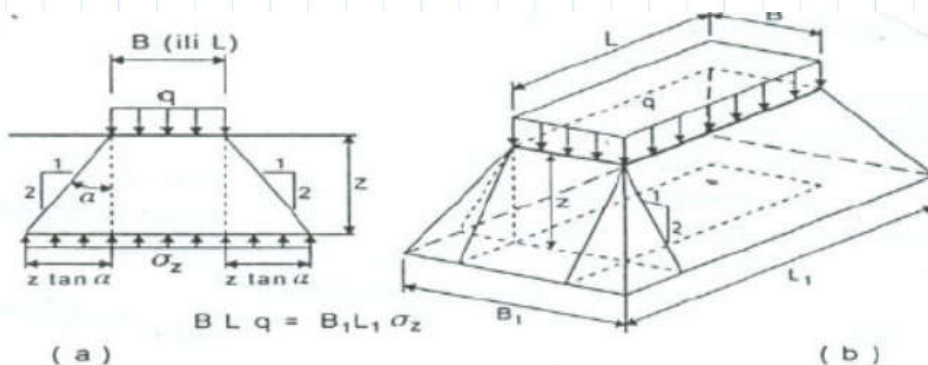
1) geostatički vertikalni naponi u tlu usled težine tla

$$\begin{aligned}
 z_0 &:= 0 \text{ m} & \sigma_{z0} &:= \gamma_t \cdot D_f = 17.5 \frac{\text{kN}}{\text{m}^2} \\
 z_1 &:= 2 \text{ m} & \sigma_{z1} &:= \sigma_{z0} + \gamma_t \cdot \Delta z = 52.5 \frac{\text{kN}}{\text{m}^2} \\
 z_2 &:= 4 \text{ m} & \sigma_{z2} &:= \sigma_{z1} + \gamma' \cdot \Delta z = 74.5 \frac{\text{kN}}{\text{m}^2} \\
 z_3 &:= 6 \text{ m} & \sigma_{z3} &:= \sigma_{z2} + \gamma' \cdot \Delta z = 96.5 \frac{\text{kN}}{\text{m}^2} \\
 z_4 &:= 8 \text{ m} & \sigma_{z4} &:= \sigma_{z3} + \gamma' \cdot \Delta z = 118.5 \frac{\text{kN}}{\text{m}^2}
 \end{aligned}$$

2) priraštaj vertikalnih napona u tlu usled spoljašnjeg opterećenja q

neto kontaktni napon $q_{neto} := q - \gamma_t \cdot D_f = 132.5 \frac{\text{kN}}{\text{m}^2}$

a) metoda ugla rasprostiranja napona $\tan\alpha=0.5$



$$\Delta\sigma_{z,i} = \frac{B \cdot L \cdot q_{neto}}{(B + 2 \cdot z_i \cdot \tan(\alpha)) \cdot (L + 2 \cdot z_i \cdot \tan(\alpha))}$$

$$\Delta\sigma_{z0} := \frac{B \cdot L \cdot q_{neto}}{(B + z_0) \cdot (L + z_0)} = 132.5 \frac{kN}{m^2}$$

$$\Delta\sigma_{z1} := \frac{B \cdot L \cdot q_{neto}}{(B + z_1) \cdot (L + z_1)} = 44.167 \frac{kN}{m^2}$$

$$\Delta\sigma_{z2} := \frac{B \cdot L \cdot q_{neto}}{(B + z_2) \cdot (L + z_2)} = 22.083 \frac{kN}{m^2}$$

$$\Delta\sigma_{z3} := \frac{B \cdot L \cdot q_{neto}}{(B + z_3) \cdot (L + z_3)} = 13.25 \frac{kN}{m^2}$$

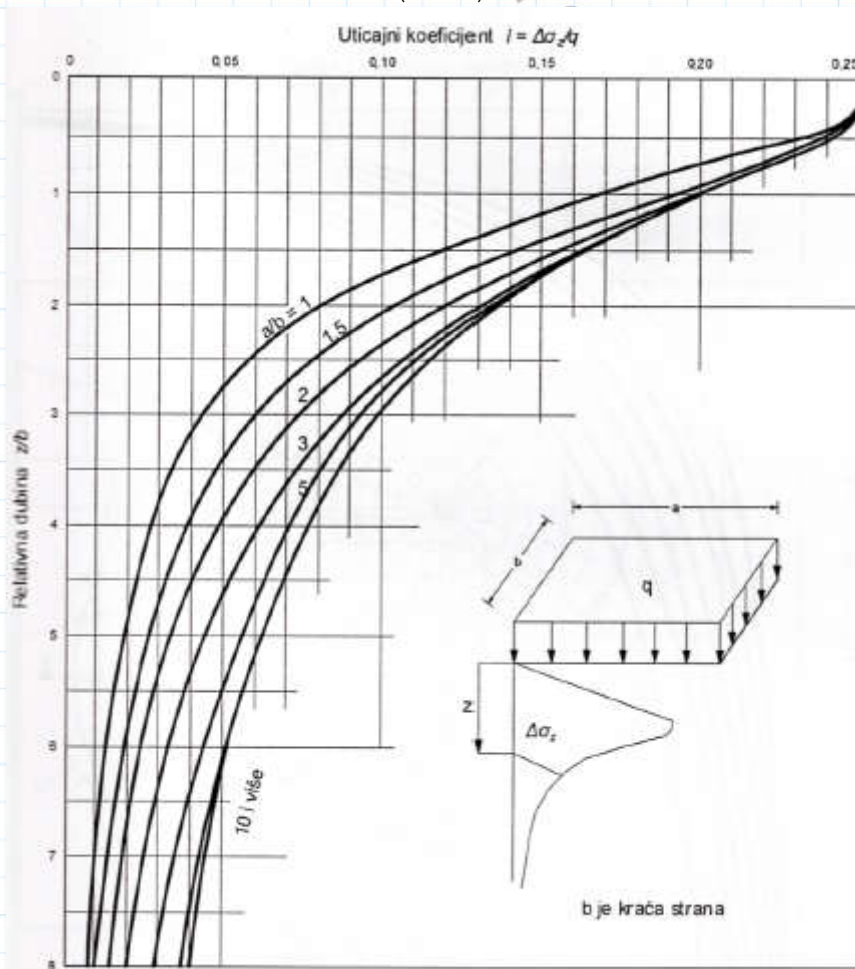
$$\Delta\sigma_{z4} := \frac{B \cdot L \cdot q_{neto}}{(B + z_4) \cdot (L + z_4)} = 8.833 \frac{kN}{m^2}$$

b) metoda Štajnbrenera

Metodom Štajnbrenera se određuje priraštaj vert. napona ispod ugaone tačke idealno savitljivog pravougaonog temelja a/b, a ≥ b.

Priraštaj napona se određuje na osnovu izraza:

$$\Delta\sigma_{z,i} = I_i \cdot q_{neto} \quad I_i = I_i \left(\frac{a}{b}, \frac{z_i}{b} \right) \text{ uticajni koef. sa dijagrama}$$



Priraštaj napona ispod sredi\u0161nje ta\u0107ke temelja se dobija podelom temelja du\u017e ose simetrije na 4 jednaka dela i sabiranjem njihovih uticaja. Odre\u011divanje prira\u0161taja naponi ispod nekih drugih ta\u0107aka temelja bi\u0107e prikazano u 2. i 3. primeru.

$$a := \frac{L}{2} = 2 \text{ m} \quad b := \frac{B}{2} = 1 \text{ m} \quad \frac{a}{b} = 2$$

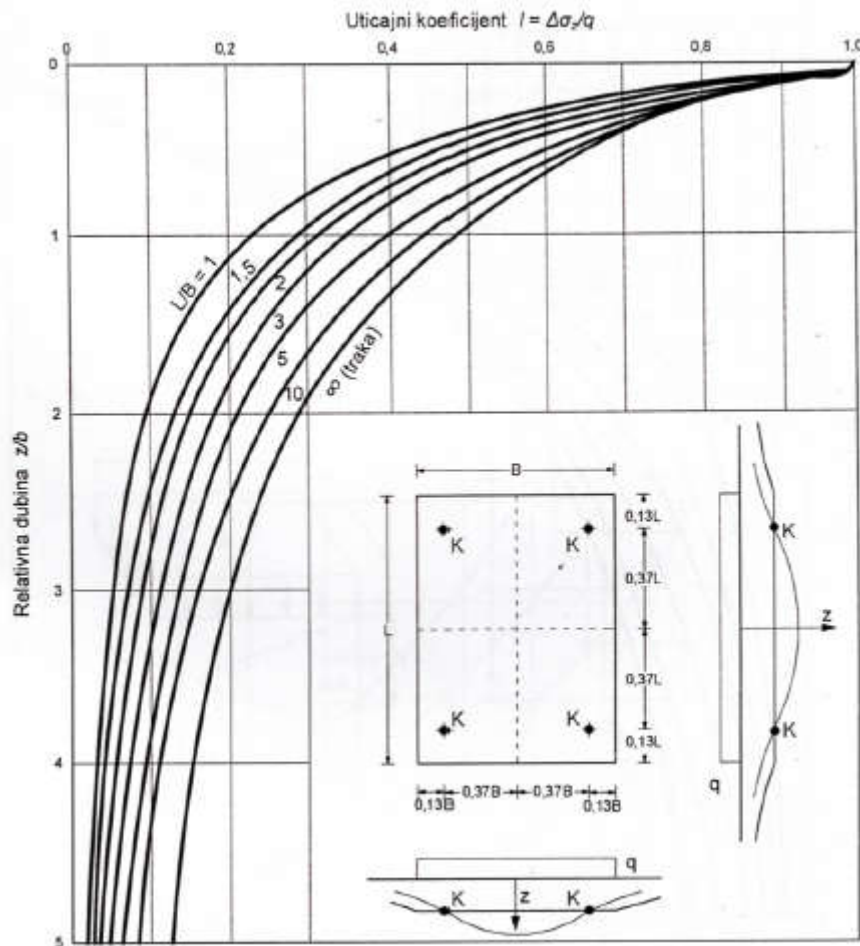
$\frac{z_0}{b} = 0$	$I_0 := 0.25$	$\Delta\sigma_{z_0.S} := 4 \cdot I_0 \cdot q_{neto} = 132.5 \frac{kN}{m^2}$	$\frac{\Delta\sigma_{z_0.S} - \Delta\sigma_{z_0}}{\Delta\sigma_{z_0}} = 0$
$\frac{z_1}{b} = 2$	$I_1 := 0.12$	$\Delta\sigma_{z_1.S} := 4 \cdot I_1 \cdot q_{neto} = 63.6 \frac{kN}{m^2}$	$\frac{\Delta\sigma_{z_1.S} - \Delta\sigma_{z_1}}{\Delta\sigma_{z_1}} = 0.44$
$\frac{z_2}{b} = 4$	$I_2 := 0.047$	$\Delta\sigma_{z_2.S} := 4 \cdot I_2 \cdot q_{neto} = 24.91 \frac{kN}{m^2}$	$\frac{\Delta\sigma_{z_2.S} - \Delta\sigma_{z_2}}{\Delta\sigma_{z_2}} = 0.128$
$\frac{z_3}{b} = 6$	$I_3 := 0.024$	$\Delta\sigma_{z_3.S} := 4 \cdot I_3 \cdot q_{neto} = 12.72 \frac{kN}{m^2}$	$\frac{\Delta\sigma_{z_3.S} - \Delta\sigma_{z_3}}{\Delta\sigma_{z_3}} = -0.04$
$\frac{z_4}{b} = 8$	$I_4 := 0.014$	$\Delta\sigma_{z_4.S} := 4 \cdot I_4 \cdot q_{neto} = 7.42 \frac{kN}{m^2}$	$\frac{\Delta\sigma_{z_4.S} - \Delta\sigma_{z_4}}{\Delta\sigma_{z_4}} = -0.16$

c) metoda Kanija

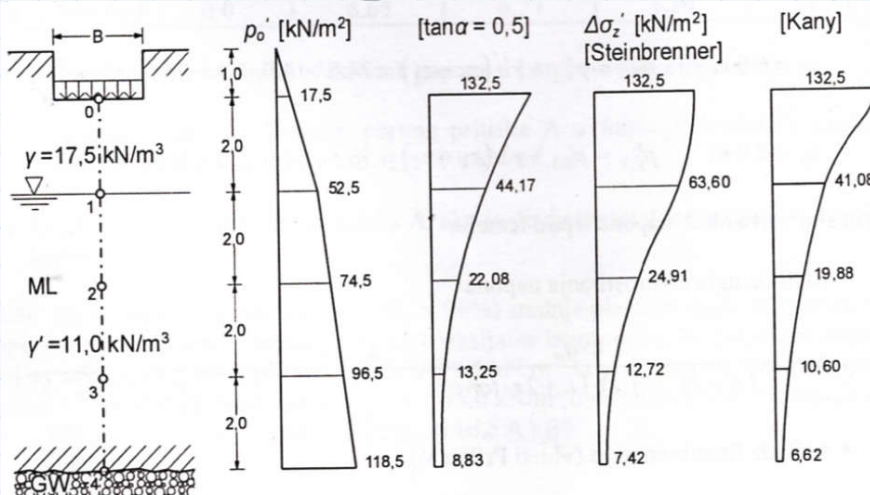
Metodom Kanija se određuje priraštaj vert. napona ispod "karakteristične" tačke idealno krutog pravougaonog temelja L/B, L ≥ B. "Karakteristična" tačka temelja je tačka ispod koje su jednaka sleganja idealno savitljivog i idealno krutog temelja i nalazi se na 0.13L i 0.13B od ugla temelja.

Priraštaj napona se određuje na osnovu izraza:

$$\Delta\sigma_{z,i} = I_i \cdot q_{neto} \quad I_i = I_i\left(\frac{L}{B}, \frac{z_i}{B}\right) \quad \text{uticajni koef. sa dijagrama}$$



$\frac{L}{B} = 2$				
$\frac{z_0}{B} = 0$	$I_0 := 1$	$\Delta\sigma_{z_0,K} := I_0 \cdot q_{neto} = 132.5 \frac{kN}{m^2}$	$\frac{\Delta\sigma_{z_0,K} - \Delta\sigma_{z_0}}{\Delta\sigma_{z_0}} = 0$	
$\frac{z_1}{B} = 1$	$I_1 := 0.31$	$\Delta\sigma_{z_1,K} := I_1 \cdot q_{neto} = 41.075 \frac{kN}{m^2}$	$\frac{\Delta\sigma_{z_1,K} - \Delta\sigma_{z_1}}{\Delta\sigma_{z_1}} = -0.07$	
$\frac{z_2}{B} = 2$	$I_2 := 0.15$	$\Delta\sigma_{z_2,K} := I_2 \cdot q_{neto} = 19.875 \frac{kN}{m^2}$	$\frac{\Delta\sigma_{z_2,K} - \Delta\sigma_{z_2}}{\Delta\sigma_{z_2}} = -0.1$	
$\frac{z_3}{B} = 3$	$I_3 := 0.08$	$\Delta\sigma_{z_3,K} := I_3 \cdot q_{neto} = 10.6 \frac{kN}{m^2}$	$\frac{\Delta\sigma_{z_3,K} - \Delta\sigma_{z_3}}{\Delta\sigma_{z_3}} = -0.2$	
$\frac{z_4}{B} = 4$	$I_4 := 0.05$	$\Delta\sigma_{z_4,K} := I_4 \cdot q_{neto} = 6.625 \frac{kN}{m^2}$	$\frac{\Delta\sigma_{z_4,K} - \Delta\sigma_{z_4}}{\Delta\sigma_{z_4}} = -0.25$	



3) vertikalne deformacije u tlu

Vertikalne deformacije u tlu se određuju za priraštaj napona određen Metodom ugla rasprostiranja napona u tlu, sa koef. proporcionalnosti M_v (modul stišljivosti tla).

$$\varepsilon_{z,i} = \frac{\Delta\sigma_{z,i}}{M_v}$$

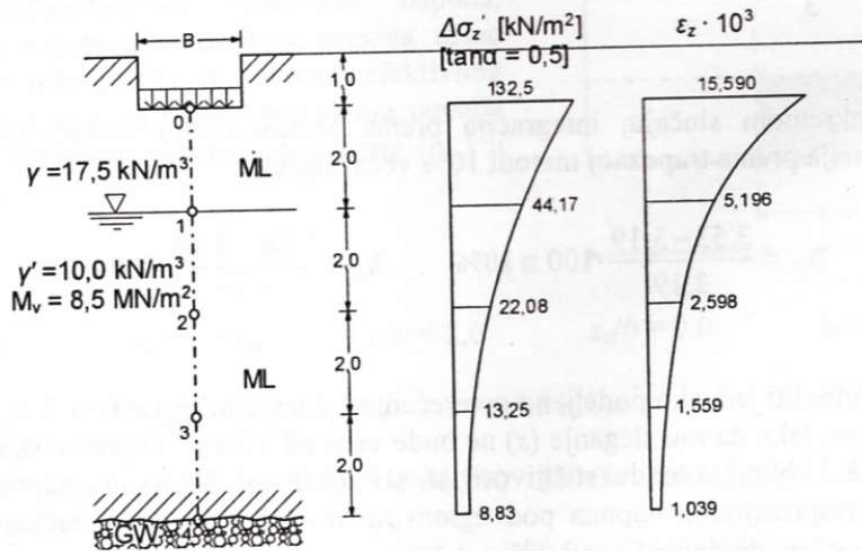
$$z_0 := 0 \text{ m} \quad \Delta\sigma_{z_0} = 132.5 \frac{kN}{m^2} \quad \varepsilon_{z_0} := \frac{\Delta\sigma_{z_0}}{M_v} = 0.01559$$

$$z_1 := 2 \text{ m} \quad \Delta\sigma_{z_1} = 44.167 \frac{kN}{m^2} \quad \varepsilon_{z_1} := \frac{\Delta\sigma_{z_1}}{M_v} = 0.0052$$

$$z_2 := 4 \text{ m} \quad \Delta\sigma_{z_2} = 22.083 \frac{kN}{m^2} \quad \varepsilon_{z_2} := \frac{\Delta\sigma_{z_2}}{M_v} = 0.0026$$

$$z_3 := 6 \text{ m} \quad \Delta\sigma_{z_3} = 13.25 \frac{kN}{m^2} \quad \varepsilon_{z_3} := \frac{\Delta\sigma_{z_3}}{M_v} = 0.00156$$

$$z_4 := 8 \text{ m} \quad \Delta\sigma_{z_4} = 8.833 \frac{kN}{m^2} \quad \varepsilon_{z_4} := \frac{\Delta\sigma_{z_4}}{M_v} = 0.00104$$



4) sleganja

Sleganje se dobija integracijom vertikalnih deformacija u tlu po dubini tla $s = \int_0^H \varepsilon_z dz = \int_0^H \frac{\Delta\sigma_z}{M_v} dz$

a) analitička integracija
$$s := \frac{q_{neto} \cdot B \cdot L}{M_v \cdot (L - B)} \cdot \ln \left(\frac{L \cdot (B + z_4)}{B \cdot (L + z_4)} \right) = 3.185 \text{ cm}$$

b) numerička integracija - trapezno pravilo

$$s_t := \frac{\Delta z}{2} \cdot (\varepsilon_{z0} + 2 \cdot \varepsilon_{z1} + 2 \cdot \varepsilon_{z2} + 2 \cdot \varepsilon_{z3} + \varepsilon_{z4}) = 3.533 \text{ cm} \quad \frac{s_t - s}{s} = 0.109$$

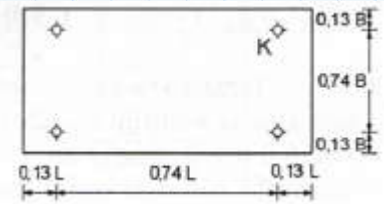
c) numerička integracija - Simpsonovo pravilo

$$s_S := \frac{\Delta z}{3} \cdot (\varepsilon_{z0} + 4 \cdot \varepsilon_{z1} + 2 \cdot \varepsilon_{z2} + 4 \cdot \varepsilon_{z3} + \varepsilon_{z4}) = 3.256 \text{ cm} \quad \frac{s_S - s}{s} = 0.022$$

2. primer

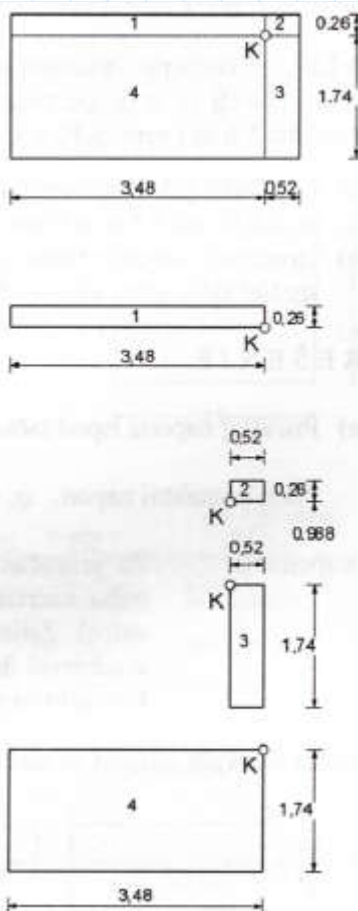
Krut pravougaoni temelj dimenzija $B/L=2/4\text{m}$ leži na sloju gline debljine $H=4\text{m}$ za koju je modul stišljivosti $tla M_v=5.0\text{MN/m}^2$. Neto kontaktni napon ispod temelja je $q_{neto}=200\text{kN/m}^2$. Sračunati priraštaj vertikalnih napona usled opterećenja duž vertikale ispod karakteristične tačke temelja K primenom metode Štajnbrenera i metode Kanija, kao i sleganje temelja na osnovu vertikalne deformacije u sredini sloja.

Temelj	$B := 2 \text{ m}$	$L := 4 \text{ m}$	$q_{neto} := 200 \frac{\text{kN}}{\text{m}^2}$	
Tlo	$H := 4 \text{ m}$	$M_v := 5 \frac{\text{MN}}{\text{m}^2}$		
Dubine	$z_0 := 0 \text{ m}$	$z_1 := 2 \text{ m}$	$z_2 := 4 \text{ m}$	$\Delta z := 2 \text{ m}$



a) metoda Štajnbrenera

$$z := z_1 = 2 \text{ m}$$



površina 1 $a_1 := 3.48 \text{ m}$ $b_1 := 0.26 \text{ m}$

$$\frac{a_1}{b_1} = 13.385 \quad \frac{z}{b_1} = 7.692 \quad I_1 := 0.040$$

površina 2 $a_2 := 0.52 \text{ m}$ $b_2 := 0.26 \text{ m}$

$$\frac{a_2}{b_2} = 2 \quad \frac{z}{b_2} = 7.692 \quad I_2 := 0.015$$

površina 3 $a_3 := 1.74 \text{ m}$ $b_3 := 0.52 \text{ m}$

$$\frac{a_3}{b_3} = 3.346 \quad \frac{z}{b_3} = 3.846 \quad I_3 := 0.066$$

površina 4 $a_4 := 3.48 \text{ m}$ $b_4 := 1.74 \text{ m}$

$$\frac{a_4}{b_4} = 2 \quad \frac{z}{b_4} = 1.149 \quad I_4 := 0.189$$

$$\Delta\sigma_z := (I_1 + I_2 + I_3 + I_4) \cdot q_{neto} = 62 \frac{\text{kN}}{\text{m}^2}$$

$$s := \frac{\Delta\sigma_z}{M_v} \cdot H = 4.96 \text{ cm}$$

b) metoda Kanija

$$z := z_1 = 2 \text{ m} \quad \frac{L}{B} = 2 \quad \frac{z}{B} = 1 \quad I := 0.31 \quad \Delta\sigma_z := I \cdot q_{neto} = 62 \frac{\text{kN}}{\text{m}^2}$$

$$s := \frac{\Delta\sigma_z}{M_v} \cdot H = 4.96 \text{ cm}$$

3. primer

Kvadratni temelj dimenzije $B=L=4\text{m}$ je fundiran na dubini $D_f=0.8\text{m}$ i opterećen centričnim opterećenjem $Q=4.24\text{MN}$. Temelj je u NPV i leži na sloju peska debljine $H=5\text{m}$ za koji je zapreminska težina tla ispod NPV $\gamma_z=18.75\text{kN/m}^3$, a modul stišljivosti tla $M_v=4.0\text{MN/m}^2$. Sračunati priraštaj vertikalnih napona usled opterećenja na dubini od 2.5m ispod tačke A na odstojanju 1m u pravcu B i u pravcu L od ugla temelja primenom metode Štajnbrenera, kao i sleganje temelja na osnovu vertikalne deformacije u sredini sloja.

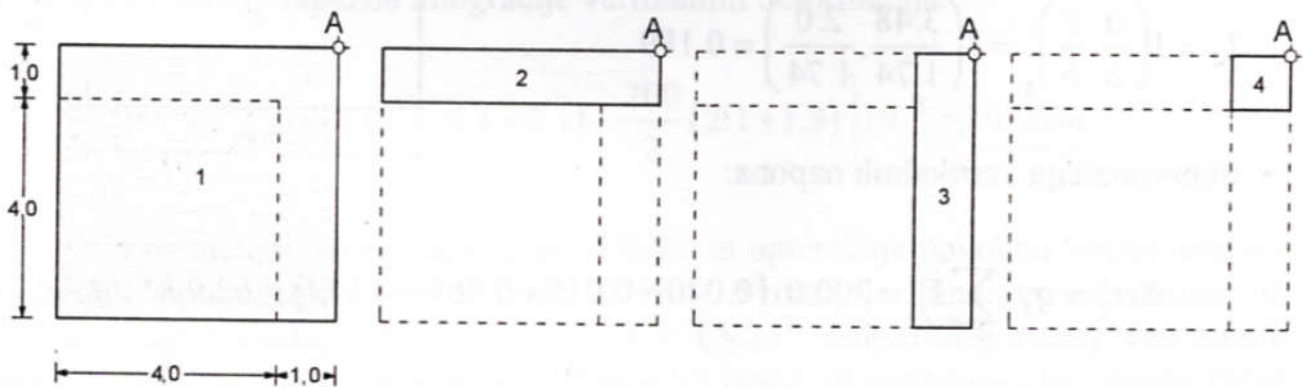
Temelj	$B := 4 \text{ m}$	$L := 4 \text{ m}$	$D_f := 0.8 \text{ m}$	$Q := 4.24 \text{ MN}$
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Tlo	$H := 5 \text{ m}$	$NPV := 0.8 \text{ m}$	$\gamma_z := 18.75 \frac{\text{kN}}{\text{m}^3}$	$M_v := 4 \frac{\text{MN}}{\text{m}^2}$
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Dubine	$z_0 := 0 \text{ m}$	$z_1 := 2.5 \text{ m}$	$z_2 := 5 \text{ m}$	$\Delta z := 2.5 \text{ m}$
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neto kontaktni napon $q_{neto} := \frac{Q}{B \cdot L} - D_f \cdot \gamma_z = 250 \frac{\text{kN}}{\text{m}^2}$

Uticajne površine za tačku A i temelj



$$z := z_1 = 2.5 \text{ m}$$

površina 1	$a_1 := 5 \text{ m}$	$b_1 := 5 \text{ m}$	$\frac{a_1}{b_1} = 1$	$\frac{z}{b_1} = 0.5$	$I_1 := 0.232$
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površina 2	$a_2 := 5 \text{ m}$	$b_2 := 1 \text{ m}$	$\frac{a_2}{b_2} = 5$	$\frac{z}{b_2} = 2.5$	$I_2 := -0.113$
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površina 3	$a_3 := 5 \text{ m}$	$b_3 := 1 \text{ m}$	$\frac{a_3}{b_3} = 5$	$\frac{z}{b_3} = 2.5$	$I_3 := -0.113$
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površina 4	$a_4 := 1 \text{ m}$	$b_4 := 1 \text{ m}$	$\frac{a_4}{b_4} = 1$	$\frac{z}{b_4} = 2.5$	$I_4 := 0.060$
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$$\Delta\sigma_z := (I_1 + I_2 + I_3 + I_4) \cdot q_{neto} = 16.5 \frac{\text{kN}}{\text{m}^2}$$

$$s := \frac{\Delta\sigma_z}{M_v} \cdot H = 2.063 \text{ cm}$$